ECE 5510 Fall 2009: Homework 10

Due: at 10:45am in the HW locker or in class, **Thursday**, December 10.

1. Y&G 11.6.1
2. A WSS signal with $R_X[k] = (\sqrt{1/2})^{|k|}$ is input to an LTI filter with $h[n] = (\sqrt{1/2})^n u[n]$. The output is Y_n.
   (a) What is $S_X(\phi)$?
   (b) What is $H(\phi)$?
   (c) What is $S_Y(\phi)$?
3. Let X_n be an i.i.d. random sequence with $\mathrm{Var}_{X_n}[X_n] = \sigma_X^2$ and zero mean. A random process Y_n is called auto-regressive with order 1 (AR-1) if it can be modeled as:

   $$Y_n = a_1 Y_{n-1} + X_n$$

   Basically, Y_n is the output of a filter, of which X_n is the input. In order to compute the most recent value of our output Y_n, we add part of the old value Y_{n-1} and the new input X_n.

   Let Y_n be an AR-1 random process, and:
   (a) Find the filter frequency response $H(\phi) = \frac{Y(\phi)}{X(\phi)}$.
   (b) Find $R_Y[k]$ using the result of (a).
   (c) Find $S_Y(\phi)$.
4. A WSS signal has the following $R_X[k]$:

   $$R_X[k] = \begin{cases} 2, & k = 0 \\ 1, & k \in \{-1, 1\} \\ 0, & o.w. \end{cases}$$

   It is input to a LTI filter with impulse response $h_n = 10\delta_n$, and the output is Y_n. What is the PSD of Y, *i.e.*, $S_Y(\phi)$?
5. Zero-mean white Gaussian WSS R.P. $N(t)$ has PSD $S_N(f) = 1$. $N(t)$ is input into an LTI system with $h(t) = 0.1e^{-t/10}u(t)$, where $u(t)$ is the unit step function. The output is $X(t)$.
   (a) Compute the PSD $S_X(f)$ of the output $X(t)$.
   (b) Compute the autocovariance $C_X(\tau)$ and the variance $\sigma^2_{X(t)}$ of $X(t)$.
   (c) Compute $P[X(0.2) > 0.1]$.
   (d) Write the joint pdf, $f_{X(1),X(6)}(x_1, x_6)$.
6. Y&G 12.1.1
7. Y&G 12.2.2. Come up with an expression, and check it in Matlab for $n = 2, 3$, and 4.