ECE 5510 Fall 2009: Homework 6

Due: at 5pm in the homework locker, Tuesday, Oct 27

1. Y&G 4.7.7

2. Y&G 4.11.2

3. (Double credit) Random variables X_1 and X_2 have joint pdf:

$$f_{X_1,X_2}(x_1, x_2) = \begin{cases} cx_1 x_2, & 0 < x_2 < x_1 < 1 \\ 0, & o.w. \end{cases}$$

   where c is a constant.

   (a) Compute the constant c in the pdf $f_{X_1,X_2}(x_1, x_2)$.

   (b) Are X_1 and X_2 independent? Explain your answer.

   (c) Compute the marginal pdf $f_{X_1}(x_1)$.

   (d) Compute the conditional pdf $f_{X_2|X_1}(x_2|0.5)$.

   (e) Compute the variance of X_1, $\text{Var}_{X_1}[X_1]$.

   (f) Compute the expectation of $X_1 X_2$, i.e., $E_{X_1,X_2}[X_1 X_2]$.

   (g) Compute the covariance matrix $C_{\mathbf{X}} = \text{Cov}(\mathbf{X})$ where $\mathbf{X} = [X_1, X_2]^T$

4. Suppose the random variables X_1 and X_2 are jointly Gaussian with mean $\mu_{X_1} = 1$ and $\mu_{X_2} = 2$, and variance $\sigma_{X_1}^2 = 1$ and $\sigma_{X_2}^2 = 4$ and correlation coefficient $\rho = 0.5$.

   (a) Write the conditional pdf $f_{X_2|X_1}(x_2|x_1)$. Specify the mean and variance of X_2 given that $X_1 = x_1$.

   (b) Let $Z = 2X_1 - 3X_2$. Find the pdf of Z.

5. Let X_1 and X_2 be i.i.d. Binomial r.v.s with parameters p and n. Let $Y = X_1 + X_2$. Find the pmf of Y. Analogous to the continuous r.v. case, the pmf of a sum Y of two independent random variables is given by the convolution,

$$P_Y(y) = \sum_x P_{X_1}(x) P_{X_2}(y - x)$$

   where $P_{X_1}(x)$ is the pmf of X_1 and $P_{X_2}(x)$ is the pmf of X_2. Also, use the fact that $\sum_{x=0}^{n} \binom{n}{x}\binom{n}{y-x} = \binom{2n}{y}$. Explain why this distribution for Y makes intuitive sense in terms of the Bernoulli trials which led to X_1 and X_2.