ECE 5510 Fall 2009: Homework 7

Due Thursday, Nov 5, at 10:45am in the homework drop box or in class.

1. Your driving route from home to work takes 20 minutes plus time spent waiting at the 10 stoplights in the route. Define the delay at each stoplight as X_i, for $i = 1, \ldots, 10$, and model X_i as i.i.d. with mean $\mu = 2$ minutes and standard deviation $\sigma = 0.5$ minute.

   (a) What is the mean and standard deviation of $Y = 20 + \sum_{i=1}^{10} X_i$, *i.e.*, your total travel time?

   (b) Your boss' route takes 30 minutes but only has 5 stoplights in route (same stoplight model as above). What is the mean and standard deviation of her total travel time?

   (c) She claims that more stoplights 'average out' your travel time, and thus you should be more reliably 'on-time' for work. Does this analysis back up this claim?

2. Y&G 5.7.7

3. Y&G 10.5.1.

4. Y&G 10.5.5.

5. Y&G 10.5.6.

6. Let $Y(t)$ be a Poisson process with arrival rate λ. Denote the first arrival time as T_1 and the second arrival time as T_2.

   (a) Find the joint pdf of (T_1, T_2). Hint: T_1 and $T_2 - T_1$ are independent.

   (b) Draw the support of (T_1, T_2). In other words, on the 2-D plot of t_1 vs. t_2, shade in the area which has $f_{T_1,T_2}(t_1, t_2) > 0$.

   (c) True or False: T_1 and T_2 are independent.

   (d) What is the probability of event A, that there was exactly one arrival in the period $[0, r)$? Hint: This is event that $\{T_1 < r\} \cap \{T_2 \geq r\}$.

   (e) Given event A (from the previous part) occurs, what is the conditional pdf of T_1, that is, $f_{T_1|A}(t_1|A)$? Interpret your results.

7. A government computer system is checked by a network admin at random times. The admin can only detect an intruder at these random times. These random times can be accurately modeled as a Poisson process with rate $\lambda = 1/10$ checks per minute.

   (a) Hacker A needs exactly 10 minutes to break into the system and download all of the system data. What is the probability that he wont be caught, *i.e.*, the admin does not check the system during the 10 minutes he is breaking into the system?

   (b) Hacker B needs 20 minutes to break into the system and download all of the system data, but she is more discrete she will only be caught if the admin checks twice while she is hacking the system. What is the probability that she wont be caught, *i.e.*, the admin does not check the system twice or more during the 20 minutes she is breaking in?