ECE 5510 Fall 2009: Homework 9

Due: at 5pm in the HW locker, **Thursday**, December 3.

1. **Mean and Autocovariance of Discrete Brownian Motion**: Let $X_0, X_1, \ldots$ be an i.i.d. Gaussian process with $\mu = 0$ and variance $T\alpha$ where T is the sampling period and α is a scale factor. The discrete Brownian motion r.p. is $Y_n = \sum_{i=0}^{n} X_i$. What are the mean function $\mu_Y[m]$ and autocovariance function $C_Y[m,k]$? Is Y_n WSS? How does this compare to the mean and autocovariance function of continuous Brownian motion derived in lecture?

2. Y&G 11.1.4

3. Y&G 11.5.1

4. Y&G 11.8.1: Hint: First, study Example 11.22.

5. Y&G 11.8.2

6. Y&G 11.8.7

7. (From Prof. Andy Yagle, U. of Michigan): A zero-mean Gaussian WSS random process $X(t)$ has covariance $C_X(\tau) = e^{-5|\tau|}$. $X(t)$ is input into an LTI system with $H(f) = 1 - \frac{15}{j(2\pi f)+20}$. The output is $Y(t)$.

   (a) Compute the power spectral density $S_Y(\omega)$ of the output $Y(t)$.

   (b) Compute the variance $\sigma^2_{Y(t)}$ of $Y(t)$.

   (c) Compute $P\left[|Y(6)| < 1\right]$.

   (d) Compute the average power in $X(t)$ between $f = 5/(2\pi)$ and $f = 5.01/(2\pi)$.