ESE 471 Spring 2021: Homework 10

1. (10 points) *Shannon Channel Capacity*: Calculate the downlink (base station to user device) channel capacity for a cellular communication link with a bandwidth of 5 MHz. Assume SRRC pulse shaping with $\alpha = 0.3$. Use the path loss exponent model to compute received power C using a transmit power of 10 W at a center frequency of 2.5 GHz, a range of 2 km, with a reference distance of 10 m, a path loss exponent $n_p = 3.5$, transmit antenna gain of 10 dB, a receive antenna gain of -3 dB, a noise equivalent temperature of $T_{eq} = 1000K$. Note that not all of this information may be necessary to solve this problem.

2. (10 points) *Decoding at a receiver*: Data bits were encoded at the transmitter with a [7,4] systematic linear block code. The S matrix is

$$S = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

   The coded bits received from the channel are:

   (a) $\mathbf{r} = [1, 1, 1, 1, 0, 0, 0]$

   (b) $\mathbf{r} = [0, 0, 0, 1, 1, 1, 0]$

   (c) $\mathbf{r} = [0, 0, 1, 1, 1, 1, 1]$

   For each received coded bit vector, decode the data and determine the data bits.

3. (10 points) For a particular $(7, 11)$ block code, the generator matrix G and syndrome matrix S are,

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

$$S = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(a) Determine the coded bits when the seven data bits are $\mathbf{d} = [1, 0, 0, 0, 0, 1, 1]$.

(b) The coded bits $[1, 0, 0, 0, 0, 0, 1, 1, 1, 0, 1]$ are received. Decode and determine the data bits sent.

4. (10 points) A transmitter with two antennas uses the Alamouti scheme: During the first symbol, it transmits s_0 out of antenna 0 and s_1 out of antenna 1. During the 2nd symbol, it transmits $-s_1^*$ out of antenna 0 and s_0^* out of antenna 1. Both symbol values are chosen from a QPSK constellation, that is, $s_0, s_1 \in \{1 + j, 1 - j, -1 + j, -1 - j\}$. The receiver has one antenna, and the channels are estimated to be $h_0 = 0.71 - 0.20j$ and $h_1 = -0.12 + 1.49j$. The receiver records $r_0 = -2.12 + 2.29j$ and $r_1 = 1.10 - 0.45j$. What were the two transmitted symbols s_0 and s_1?

5. (10 points) A 2x2 MIMO system is transmitting two symbols $\mathbf{s} = [s_0, s_1]^T$ that are chosen from a QPSK constellation, that is, $s_0, s_1 \in \{1 + j, 1 - j, -1 + j, -1 - j\}$. The received signal $\mathbf{x} = H\mathbf{s} = [-4.2 + 5.7j, -1.4 - 1.9j]^T$. The receiver measures the channel to be:

$$H = \begin{bmatrix} 0.3 + 3.6j & -0.4 - 1.3j \\ -1.3 + 2.8j & 0.3 + 3.0j \end{bmatrix}.$$

What were the two transmitted symbols s_0 and s_1?