ESE 471 Spring 2021: Homework 2

1. Given symbol set $\mathcal{S} = \{s_0(t), s_1(t), \ldots, s_{M-1}(t)\}$ and each symbol is in span of the orthonormal basis $\mathcal{B} = \{\phi_0(t), \phi_1(t), \ldots, \phi_{K-1}(t)\}$, prove that the energy of a signal $s_m(t)$ calculated in the time domain and in signal space are identical, *i.e.*,

$$\int_{-\infty}^{\infty} |s_m(t)|^2 dt = \sum_{k=0}^{K-1} |a_{m,k}|^2$$

where $a_{m,k} = \langle s_m(t), \phi_k(t) \rangle$. Hint: Note that $s_m(t) = \sum_{k=0}^{K-1} a_{m,k}\phi_k(t)$ because it is in the span of the orthonormal basis.

2. Consider the signal set of transition position-modulated signals $s_0(t), s_1(t), s_2(t), s_3(t)$ in which the switch between 1 and -1 occurs at time $i+1$,

$$s_i(t) = \begin{cases} 1, & 0 \le t \le i+1 \\ -1, & i+1 < t < 4 \\ 0, & o.w. \end{cases}$$

The orthonormal basis functions are the Walsh-Hadamard functions shown in Figure 1 (the same as in Homework 1). Compute the signal space vectors $\mathbf{s}_m = [a_{m,0}, \ldots, a_{m,3}]^T$ for $m = 0, 1, 2, 3$.

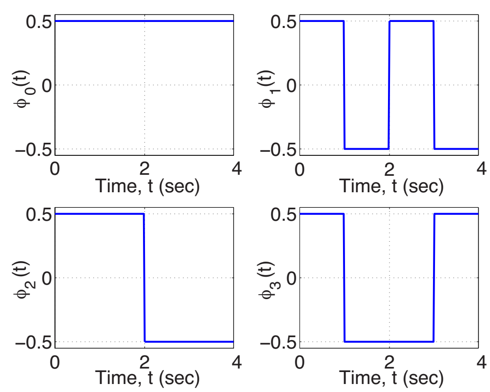

Figure 1: Walsh-Hadamard orthonormal basis.

3. Rice 2.17. Copied here: Consider the signal $x(t) = A\cos(\omega_0 t + \theta)$. (a) Compute the Fourier transform $X(j\omega)$. (b) Compute the Fourier transform $X(f)$.

4. Rice 2.25. Copied here: The 3-dB bandwidth B_{3dB} of a baseband signal $x(t)$ is defined using the Fourier transform $X(f)$: B_{3dB} is the value of f for which $|X(f)|^2 = |X(0)|^2/2$. Determine B_{3dB} for each of the following signals.

   (a) $x(t) = e^{-at}u(t)$

   (b) $x(t) = e^{-a|t|}$

   (c) $x(t) = e^{-\pi t^2}$