ESE 471 Spring 2021: Homework 5

1. (20 points) In Lecture 10, when deriving the threshold for a 1-D binary detector, we assumed that the noise N had equal variance regardless of whether $s_0(t)$ or $s_1(t)$ was sent. Assume instead that the transmitter is faulty and adds noise to the transmitted signal whenever $s_1(t)$ is sent. That is the receiver measures X as either:

$$\begin{aligned} H_0: \quad & X = a_0 + N_0 \\ H_1: \quad & X = a_1 + N_1 \end{aligned}$$

   where $a_0 = 0$, $a_1 = 1$, and N_0 and N_1 are both zero-mean Gaussian noise, in this case, where N_0 has variance 1 and N_1 has variance 2. Assume that $P[H_0] = P[H_1] = 1/2$. What is the optimal decision region R_0 (the region in which we decide H_0)?

2. (10 points) Consider a binary 1-D communication system modeled as $H_0 : X = a_0 + W$, or $H_1 : X = a_1 + W$, where $a_0 = -1$, $a_1 = +1$, and W is zero mean Gaussian with variance $\sigma_W^2 = 0.3$. First, assume $P[H_0] = P[H_1] = 1/2$.

   (a) What is the optimal decision threshold γ?

   (b) What is the probability of error for the optimal receiver?

   For the next two parts, assume that the signals are not equally probable, and instead, $P[H_0] = 0.2$ and $P[H_1] = 0.8$.

   (c) What is the optimal decision threshold γ?

   (d) What is the probability of error for the optimal receiver?

3. **Optimal Detection Verification** (20 points): Continue to consider the above communication system, with $H_0 : X = a_0 + W$, or $H_1 : X = a_1 + W$, where $a_0 = -1$, $a_1 = +1$, and W is zero mean Gaussian with variance $\sigma_W^2 = 0.3$, and $P[H_0] = 0.2$ and $P[H_1] = 0.8$. In this problem, you will verify experimentally that the optimal decision threshold minimizes the probability of error. In Matlab or Python (or language of your choice),

   (a) Create a vector of 5000 bits sent by the transmitter, each bit b_l is equal to 0 with probability $P[H_0]$ and equal to 1 with probability $P[H_1]$. Show Matlab output of the histogram of b_l to verify its pmf.

   (b) Generate X as follows: When $b_l = 0$, let $X = a_0 + W$. When $b_l = 1$, let $X = a_1 + W$. W is generated with σ_W`*randn` in Matlab. In Python, use `random.gauss(0,`σ_W`)` (after having done an `import random`). No output is needed for this part.

   (c) Create a vector listing possible threshold values, `etaList = -2:0.01:2` in Matlab or `numpy.arange(-2, 2.01, 0.01)` in Python. For each value η in the list, calculate the bit error rate using a threshold of η. Specifically, make bit decisions by the detection rule, $X \underset{H_0}{\overset{H_1}{\gtrless}} \eta$, Denoting these bit decisions $\hat{b}_l$, count the experimental bit error rate for that η. Plot the experimental bit error rate vs. `etaList`. Draw a vertical line on the plot at $\eta = \gamma$, where γ is the optimal threshold (see Problem 2(c)), and show that the error rate is approximately minimum at the optimal threshold.

   (d) Do parts (a) through (c) again for the case when H_0 and H_1 are equally probable. The only output necessary is the final plot of experimental bit error rate vs. `etaList`.