ESE 471 Spring 2021: Homework 6

1. (20 points total, 4 points each) Consider the $M = 3$ PAM system that has symbol values $\mathbf{a}_0 = -2A$, $\mathbf{a}_1 = 0$, and $\mathbf{a}_2 = 2A$. While such a PAM system does not exist in practice, we are using it in this problem to allow you to demonstrate your ability to analyze PAM probability of error. In such a system, we'd use a trinary digit to measure information. A trinary digit takes value 0, 1, or 2 (instead of a binary digit of value 0 or 1). We measure trinary digits as $\log_3 M$. The average trinary digit energy $\mathcal{E}_{tav}$ is the average symbol energy divided by $\log_3 M$. Assume that each symbol is equally likely.

   (a) What is A in terms of $\mathcal{E}_{tav}$?

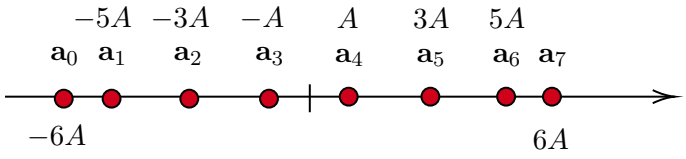
   (b) What are the optimal decision thresholds?

   (c) What is $P\left[\text{Symbol Error}|H_0\right]$?

   (d) What is $P\left[\text{Symbol Error}|H_1\right]$?

   (e) What is the average probability of symbol error for the $M = 3$ PAM system, $P\left[\text{Symbol Error}\right]$, as a function of $\mathcal{E}_{tav}/N_0$?

2. (20 points total, 5 points each) Why do PAM systems use equally-spaced symbol amplitudes? To evaluate this, let's evaluate the following non-uniform $M = 8$ PAM modulation here:



   The $\mathbf{a}_0$ and $\mathbf{a}_7$ symbols have been moved closer to 0 to reduce their contribution to the average energy. Assume that symbols are equally likely.

   (a) What are the optimal decision thresholds?

   (b) What is the average symbol energy as a function of A?

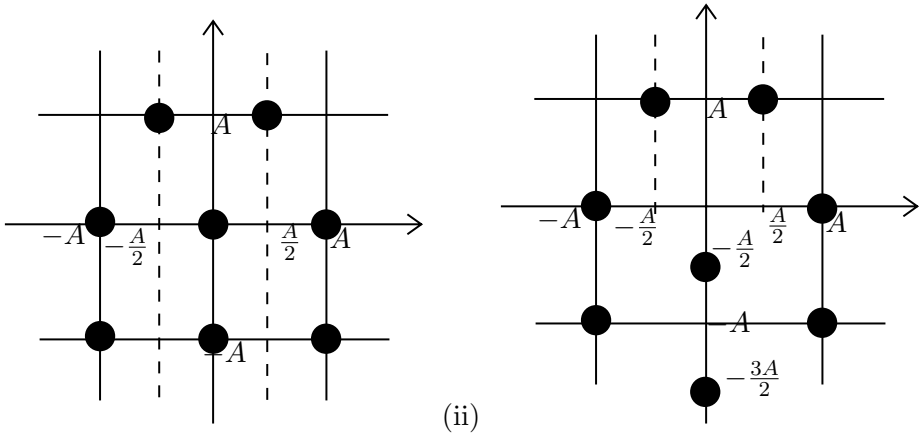
   (c) Compute the average probability of symbol error for the $M = 8$ PAM system, $P\left[\text{Symbol Error}\right]$, as a function of $\mathcal{E}_b/N_0$.

   (d) In a graphing program (Matlab/Python), plot the probability of symbol error as a function of $\mathcal{E}_b/N_0$ for your answer in (b) as well as the probability of symbol error for normal (equally-spaced symbol amplitude) 8-PAM. Use a range of $\mathcal{E}_b/N_0$ from 10 to 100 (linear) or 10-20 on a dB scale, and include your plot in your submission. Which one is better?

3. (10 points total, 5 points each) For each of the QAM constellations in the Figure below,

   (a) Find the average bit energy as a function of A.

   (b) Draw the boundaries between the decision regions.


A
−A
−A/2
A/2
A
−A
(i)
A
−A
−A/2
A/2
A
−A/2
−A
−3A/2
(ii)