ESE 471 Spring 2021: Homework 7

1. (10 points) Consider the $M = 9$ QAM system in Figure 1. This is a two-dimensional version of the $M = 3$ PAM system from Homework 6 Question 1. In this $M = 9$ QAM system, we are using a trinary digit to measure information. A trinary digit takes value 0, 1, or 2 (instead of a binary digit of value 0 or 1). We measure trinary digits as $\log_3 M$ trinary digits per symbol. The average trinary digit energy $\mathcal{E}_{tav}$ is the average symbol energy divided by $\log_3 M$. Answer the following questions for this $M = 9$ QAM system.

   (a) What is A in terms of $\mathcal{E}_{tav}$?

   (b) What is the probability of symbol error as a function of $\mathcal{E}_{tav}/N_0$? Find an exact expression, and the union bound.

   (c) What is the probability of trinary digit error as a function of $\mathcal{E}_{tav}/N_0$?

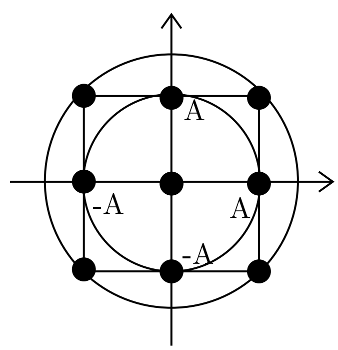

Figure 1: The $M = 9$ QAM constellation for problem 1.

2. (30 points total, 15 for each constellation) Find expressions for the probability of symbol error for each of the $M = 8$ constellation diagrams in Figure 1 [c] and [d] of the Lecture 15 notes. Find:

   (a) (5 points) The average energy per symbol $\mathcal{E}_s$ in terms of A.

   (b) (5 points) The union bound on the probability of symbol error.

   (c) (5 points) The nearest neighbor approximation for the probability of symbol error.

   The answers should be in terms of $\mathcal{E}_s$, not $\mathcal{E}_b$.

3. (10 points) Two ways to find $P\,[\text{error}]$ for QPSK: QPSK can be thought of as either a square QAM constellation with four points or as 4-ary PSK.

   (a) Using the technique for analyzing square QAM constellations outlined in the lectures, derive the probability of bit error for QPSK.

   (b) Apply the general exact expression for probability of error for MPSK to the special case $M = 4$ to derive an expression for the probability of bit error for QPSK and show that it is the same result as (a).