ESE 471 Spring 2021: Homework 9

1. (20 points) Design a new wireless communication system for a high data rate VR/AR system that requires a bit rate of 100 Mbps with the following constraints:

   - The system will operate in an band of at most 40 MHz bandwidth at a center frequency of 60 GHz.
   - Assume a transmit power $P_T = 10$ dBm, the antenna gains are $G_t = 1$ dB and $G_r = 7$ dB.
   - The path loss is given by the path loss exponent model with a reference distance of 1 meter, and a path loss exponent of 2.5.
   - Design with a SRRC pulse with an $\alpha \geq 0.3$.
   - The range must be at least 10 m.
   - The receiver has a noise figure $F = 7$ dB, operating with a $T_0 = 290$ K.
   - It must have a bit error rate of at most 1×10^{-3} to have a sufficient quality of interaction with the user.

   Specify a system solution that meets these requirements, including M, modulation, α, bandwidth, and bit rate. You may explore design options either by hand or a link budget spreadsheet (mine, for example), but if you use a spreadsheet please work out the performance of your final design by hand to allow us to check your work.

2. (20 points) A communication system is designed to communicate commercial airplane radar information $\{X_i\}$ back to an air traffic control tower. The radar measures either $X_i = 1$ for airplane present in a particular region of focus for this radar or $X_i = 0$ for no airplane present at time i. The radar makes a new measurement 5 times per second. Past measurements tell us that the $P[X_i = 1] = 0.01$ at a particular instant. Here we assume that $\{X_i\}$ are independent and identically distributed.

   (a) What is the entropy rate of this information source, in bits per second?

   (b) Consider the system that simply sends the one bit value of X_i after each measurement i. What is the bit rate of this communication system, in bits per second? (This is not a trick question, it should not involve much work.)

   (c) Consider this simple lossless compression scheme. Five measurements (over one second) $X_i, \ldots, X_{i+4}$, are are communicated simultaneously as follows. If all five are '0', then only a single bit '0' is transmitted. If any of the five is 1, then a '1' is transmitted, followed by the five measurements (each one bit) themselves, for a total of 6 bits. What is the average bit rate of this compression scheme, in bits per second?

3. (10 points) Let the random process X_i be an i.i.d. sequence of uniformly distributed random variables, each uniformly distributed on a finite event set with size K (*i.e.*$|S_X| = K$).

   (a) Find the entropy rate H of random process $\{X_i\}$.

   (b) What minimum rate R_b, in bits per source output, will be necessary to encode the source $\{X_i\}$ without error?

   (c) Does H increase or decrease with K? Why does it make sense? (Or why does this not make sense?)